JUTLP
Journal of University Teaching & Learning Practice

How would future mathematicians embrace digital literacy? A TAM model-based analysis

Dr Zhenyu Sun[a], Associate Professor Ruixue Yuan[b], and Professor Xuezhi Liu[b]
[a] University of Chinese Academy of Sciences, China; [b] Beijing University of Chemical Technology, China

Abstract

The widespread incorporation of digital technologies in higher education requires a thorough understanding of how these tools are embraced and applied, especially in the field of mathematics. This study employs the Technology Acceptance Model (TAM) to examine the adoption of digital tools in university-level mathematics education, focusing on their model-related constructs. Through a detailed analysis of innovative higher educational initiatives like the PAL course series at Carnegie Mellon University and the Xena Project at Imperial College London, our findings indicate that digital literacy significantly enhances conceptual understanding, facilitates advanced research, and improves problem-solving capabilities among students majoring mathematics. The empirical evidence strongly supports the predictive power of the TAM in this educational context, suggesting that increasing the perceived ease of use and usefulness of digital tools can substantially improve their acceptance. These insights underscore the critical role of digital literacy in reshaping educational landscapes and highlight the necessity for educational policies to encourage the development and integration of effective digital tools in mathematical education, thereby preparing future mathematicians for a digitally integrated academic and professional environment.

Editors

Section: Special Issue
Senior Editor: Dr Cassandra Colvin
Guest Editor: Dr Michael O'Dea

Publication

Submission: 30 December 2024
Accepted: 14 August 2024
Published Online: 20 September 2024

Keywords

Higher Mathematics Education, Technology Acceptance Model, Digital Literacy, Formal Mathematics, Proof Assistant

Citation

Sun, Z., Yuan R., & Liu X. (2024). How would future mathematicians embrace digital literacy? A TAM model-based analysis. *Journal of University Teaching and Learning Practice, Advanced Online Publication* https://doi.org/10.53761/x1m1ja93 .

Practitioner Notes

1. Educators should consider integrating digital literacy into their curricula to facilitate deeper conceptual understanding and more effective problem-solving skills among students.
2. Universities and educational technology developers should focus on creating user-friendly digital resources that clearly demonstrate their practical benefits in educational settings.
3. Incorporating digital tools like theorem provers (interactive or automated) into mathematics courses requires thoughtful curriculum design. This integration should aim to gradually introduce students to complex tools through structured activities that build on existing knowledge and skills.
4. Effective use of digital tools in mathematics education necessitates ongoing professional development for educators. Training programs should focus on both the technological aspects of these tools and pedagogical strategies for their implementation.
5. The advancement of digital tools in mathematics education benefits from active research and collaborative efforts among academics. Institutions should encourage participation in projects that explore and expand the use of such technologies, similar to the PAL courses and the Xena Project.

Introduction

The landscape of mathematical education is undergoing a profound transformation, precipitated by the rapid advancement of digital technologies. As digital tools become increasingly pervasive across various facets of life, their integration into educational frameworks has become a pivotal focus of contemporary educational reform. We explore the critical role of digital literacy within university-level mathematics education, a topic of paramount relevance given the challenges and opportunities presented by the digital era.

Traditionally, mathematics has been taught through methods that emphasize theoretical understanding and manual problem-solving skills. However, the swift evolution of technology necessitates a paradigm shift towards the incorporation of digital tools that enhance learning outcomes and comprehension. This transition represents not merely a trend but a fundamental shift in achieving educational goals in higher mathematics. The integration of platforms such as mathematical software and online resources into the learning environment promises to redefine the acquisition and application of mathematical knowledge.

Despite the clear benefits, the adoption of digital tools in mathematics education often encounters resistance due to a prevailing lack of understanding and acceptance among educators and institutions. This resistance, often rooted in a traditional mindset, undervalues the potential of technology-enhanced learning. While certain studies highlight the benefits of digital literacy in enhancing educational outcomes (Anuratha, 2020), there is a significant gap in systematically understanding how these tools can be effectively integrated into tertiary mathematics education.

To bridge this critical gap, our study employs the Technology Acceptance Model (TAM), developed by Fred D. Davis (Davis, 1989), which provides a robust framework for understanding the factors that influence the acceptance of new technologies. Moreover, we integrate Actor-Network Theory (ANT) (Latour, 2005) to provide a more comprehensive analysis of the complex interrelations among various actors involved in the adoption of digital tools in university-level and more advanced mathematics education.

This research aims to address two main issues: firstly, to identify the key determinants influencing educators' and students' acceptance and use of digital tools in university-level mathematics education; and secondly, to evaluate the impact of these tools on educational practices and learning outcomes. Through a mixed-methods approach that combines text interpretation of TAM constructs and qualitative case studies enhanced by ANT insights, this study endeavors to provide a theoretical and empirical foundation for strategies that facilitate the effective integration of digital tools into mathematical education, thus enhancing both teaching quality and student learning experiences.

Our study aims to pave the way for further investigation into how digital literacy can profoundly transform educational practices. It underscores the need for educational policies that promote the development and integration of effective digital tools in mathematical education. Such initiatives are essential for equipping future mathematicians with the skills necessary to thrive in a digitally integrated academic and professional landscape.

Literature

Role and Acceptance of Digital Tools in Education

Digital tools extend beyond simplifying calculations to transform pedagogical strategies, enhancing interactive learning, and promoting collaboration among students (Sinclair & Yerushalmy, 2016). These tools foster a more active and engaging learning environment, allowing for deeper exploration of mathematical concepts. The effectiveness of these tools in fostering educational outcomes is further supported by research examining the perceptions of technology use in mathematics by university students (Zogheib et al., 2015).

However, not all researchers agree on the positive impact of digital literacy in education. For instance, authors explore the application of artificial intelligence in teaching and learning in higher education, noting that despite the rapid development of AI technology, there is currently insufficient evidence to support its practical effectiveness in teaching (O'Dea & O'Dea, 2023).

Recent Technological Advancements in Mathematics Education

The integration of digital tools in mathematics education signifies a major paradigm shift, transitioning from traditional pedagogical methods to innovative digital methodologies. One of the most notable advancements is the foundational framework of homotopy type theory (HoTT), which merges mathematical reasoning with programming language elements, exemplifying this transformation (Voevodsky, 2006; Awodey, 2014). Because this new foundation is heavily reliant on computer science and technology, some mathematics educators are now working on developing innovative university-level educational environments based on it (Bezem et al., 2022).

Technology Acceptance Model (TAM) in Educational Settings

The Technology Acceptance Model provides a robust framework for understanding how perceptions of usefulness and ease of use influence the adoption of technology in educational settings (Davis, 1989). This model has been adapted to explore various educational levels and settings, such as the acceptance of digital mathematics games by elementary teachers, highlighting the importance of perceived usefulness and ease of use (Yeo, Rutherford, & Campbell, 2022). Additionally, the influence of teachers' digital competence on their acceptance of technology is emphasized in vocational education contexts (Antonietti, Cattaneo, & Amenduni, 2022).

The ongoing development of TAM in educational technology research includes conceptualization and extension studies that provide a deeper understanding of the factors influencing technology integration. Researchers use a grounded theory approach to expand on TAM findings within secondary-school mathematics education (Ince-Muslu & Erduran, 2021), while others examine the antecedents of Information and Communication Technologies (ICT) adoption across educational settings, utilizing an extended TAM (Teeroovengadum, Heeraman, & Jugurnath, 2017).

Empirical Evidence Supporting Digital Tool Integration

Empirical studies provide valuable insights into the practical implications of integrating digital tools in educational settings. For instance, the impact of ICT on mathematics teachers' acceptance of technology showcases key factors affecting their usage (Soydaş, 2023). Moreover, the assessment of self-efficacy among Chinese primary mathematics teachers using digital tools highlights significant effects on student learning outcomes and teacher efficacy (Li, 2024). Extensive research also measures the acceptance of digital tools in open and distance learning environments, identifying critical factors that influence their acceptance and use (Noh et al., 2023).

Method

Study Design and Theoretical Framework

This research employs a text-based case study methodology to explore the integration of digital tools in university-level mathematics education, focusing specifically on the Pure and Applied Logic (PAL) courses at Carnegie Mellon University and the Xena Project at Imperial College London.

The choice of a text-based case study is appropriate for in-depth exploration within real-life contexts where the phenomenon and context are intricately linked; while the the rarity of samples in the worldwide environment, the inaccessibility of direct classroom observations or experiments, and the diversity of text types also served as important considerations.

The theoretical foundation of the study is based on the Technology Acceptance Model, which is utilized to examine how perceived usefulness (PU), perceived ease of use (PEOU) and other model constructs influence the acceptance and integration of technology by educators and students.

TAM's Evolution and Its Relevance to Mathematics Education

TAM has evolved through integrations with other behavioral theories, enhancing its applicability to diverse contexts. Notably, the integration of TAM with the Theory of Planned Behavior (TPB) and the Unified Theory of Acceptance and Use of Technology (UTAUT) has deepened insights into user behavior in technology adoption. Modifications such as incorporating factors like social influence, system design, and user experience have broadened TAM's scope, making it highly relevant for analyzing digital tools in educational settings.

In this study, TAM provides a structured framework to assess:

- Transition to Digital Platforms: How digital tools are perceived in terms of their utility and ease of integration into existing teaching practices.
- Complexity in Mathematics Education: The capability of digital tools to simplify or make accessible complex mathematical concepts without compromising educational integrity.
- Pedagogical Innovations: The impact of digital transformation on teaching methodologies and curriculum development.

Actor-Network Theory (ANT) Application

In addition to the Technology Acceptance Model, our study incorporates the Actor-Network Theory to provide a comprehensive framework for analyzing the complex interrelations between various actors involved in the adoption of digital tools in university-level mathematics education. ANT is used in this research to examine the roles and influences of human and non-human entities that collectively affect the integration and utilization of digital technologies.

More concretely, ANT is applied to identify and map the network of relationships that influence the acceptance and use of digital tools. This includes the interactions between students, educators, technological tools (like the Lean theorem prover), and institutional policies. By analyzing these interactions, ANT helps to uncover how various actors contribute to the stabilization and acceptance of digital tools within educational practices.

To ensure a holistic understanding of technology acceptance, ANT is integrated with TAM to explore not only the perceptions of usefulness and ease of use as influenced by individual actors but also how these perceptions are shaped through the network of interactions. This integrated approach allows for a deeper understanding of the socio-technical dynamics that influence technology adoption in educational settings.

Data Sources

The study leverages publicly available data from multiple sources:

- Blogs and lectures: Blog entries from leaders and core practitioners of the relevant educational project, as well as slides from public lectures, are freely accessible from the project's homepage.
- Online Community Discussions: Analysis of Zulip chat platform discussions where the usage, benefits, and challenges of digital tools like Lean are debated.
- Publicly Available Resources: Examination of materials from conferences and open educational resources that discuss the adoption and impact of digital tools in mathematics education.

Ethical Considerations

All research data comes from publicly available sources, including academic publications, blogs, lectures, and openly shared educational evaluations. Most crucially, the open-source nature of the HoTT project and Lean software and their communities discussions further align with the study's ethical commitment to using public domain information without infringing on private data.

Results

This section aims to synthesize the results from two distinct case studies at Carnegie Mellon University (CMU) and Imperial College London (ICL) concerning the integration of digital tools in mathematics education. By merging the analysis of these case studies with a direct comparison, we intend to highlight the key similarities and differences in digital tool adoption and their impacts. This unified approach enables a deeper understanding of how contextual factors at each institution influence the effectiveness and acceptance of technology in educational settings.

The CMU case study explores the use of the Lean theorem prover to enhance the teaching of formal logic and proof techniques in mathematics courses, emphasizing the practical application of advanced digital tools. In contrast, the ICL case study focuses on the Xena Project, which integrates similar digital tools to foster interactive learning and engagement in actual research-level mathematics. Both initiatives are evaluated through the lens of the Technology Acceptance Model, providing insights into their usability and practical benefits. This comparison not only reveals the unique strategies and outcomes at each institution but also contributes to the broader discourse on digital literacy in higher education.

Detailed Case Analysis

CMU Case Study

The research primarily draws upon records of public lectures given by professor Jeremy Avigad, the PAL course project leader, on various occasions (Avigad, 2021a; 2021b; 2022; 2024).

Context and Implementation

At Carnegie Mellon University, the integration of digital tools into logic and mathematics education primarily revolves around the Lean theorem prover, which has been implemented as a central component of the curriculum in several courses ranging from undergraduate to advanced graduate levels. This implementation is part of a broader initiative to enhance the analytical capabilities of students and to provide them with hands-on experience in formalized mathematics and automated proof verification.

The use of Lean at CMU is situated within the Department of Philosophy and the Department of Mathematical Sciences, both have a longstanding tradition of incorporating computational elements into its curriculum. The adoption of Lean is driven by the departments' goal to fuse traditional mathematical teaching methods with modern computational tools, thereby preparing students for the increasingly digital nature of mathematical research and application. This strategic integration also addresses the growing demand for graduates who are proficient in both theoretical and applied aspects of mathematics.

Lean has been incorporated into various courses, including:

- Introduction to Theoretical Mathematics: where students first encounter formal proof techniques. Lean is used to introduce students to the rigor of mathematical proofs, allowing them to verify their proofs interactively.
- Advanced Logic and Set Theory: which uses Lean extensively to explore complex logical frameworks and set theoretical concepts. Students engage with Lean to construct and verify proofs that are otherwise too intricate to handle without computational assistance.
- Research Projects and Thesis Work: where graduate students utilize Lean to formalize and validate their research findings, often in collaboration with their supervisors, who are also proficient in using the tool.

Specific examples of Lean's concrete application include:

- Formalizing Proofs of Classical Theorems: Students are tasked with translating proofs of well-known mathematical theorems into Lean's formal language, enhancing their understanding of both the proofs themselves and the process of formalization.
- Interactive Proof Competitions: Organized within the department, these competitions challenge students to solve complex mathematical problems using Lean, promoting a deeper engagement with the tool and fostering a competitive yet collaborative learning environment.

The scope of tool usage extends beyond classroom settings, with many faculty members adopting Lean for collaborative research projects. This not only enhances the research output of the departments by ensuring the correctness of proofs but also creates a vibrant academic community centered around digital tool proficiency and innovation in mathematics. Through these implementations, CMU has established a robust model of integrating digital tools into mathematics education, setting a benchmark for similar initiatives in other institutions.

TAM analysis

Table 1

TAM ANALYSIS OF THE CMU CASE

Model Construct	Analysis
External Variables	External variables such as community support and resource availability played crucial roles in shaping the acceptance and usage of Lean at CMU. The strong backing from the Lean community, including access to a broad range of libraries and collaborative projects, enhanced the resource environment for both students and instructors. This support system not only eased the integration of Lean into the curriculum but also fostered a collaborative and resource-rich learning atmosphere, which was pivotal in enhancing the perceived ease of use and usefulness of Lean.
Perceived Usefulness (PU)	At CMU, the use of Lean in the Logic and Mechanized Reasoning courses has significantly enhanced students' understanding of formal methods and logical reasoning. The integration of Lean has allowed students to engage deeply with

	complex mathematical proofs, providing them with a clear, structured approach to learning. This methodological clarity is recognized as highly beneficial for mastering rigorous academic content, which in turn, improves their professional preparedness and attractiveness to potential employers in fields that value analytical and computational skills.
Perceived Ease of Use (PEOU)	Initially, students faced challenges with the technical complexity and the steep learning curve of Lean. However, over time, the structured nature of Lean's environment, combined with instant feedback on proof attempts, significantly reduced these barriers. This ongoing interaction and the gradual acclimatization to Lean's functionalities have improved its perceived ease of use. The university's provision of comprehensive support resources, including detailed documentation and community forums, further facilitated this process, making the tool more accessible and easier to integrate into daily academic activities.
Attitude Toward Using (A)	The overall attitude toward using Lean at CMU has been increasingly positive, influenced significantly by the interactive and engaging nature of the tool. The ability to receive immediate feedback and visually track the logical progression of proofs has made learning more dynamic and appealing. These positive experiences have fostered a favorable attitude towards Lean, encouraging both students and faculty to advocate for and continue using the tool in educational settings.
Behavioral Intention to Use (BI)	The behavioral intention to use Lean among CMU students and faculty is high. This intention is driven by the recognition of Lean's direct applicability to academic success and career advancement. The skills developed through using Lean—such as logical reasoning, problem-solving, and functional programming—are highly valued in various professional fields, thus motivating students to master the tool and incorporate it into their future professional toolkit.
Actual System Use	The actual use of Lean at CMU has expanded across various levels of the mathematics curriculum, from introductory courses to advanced graduate research. This widespread adoption is indicative of the tool's integration into the academic culture at CMU, where students and faculty utilize Lean not just for course requirements but also in research projects and independent learning endeavors. The increasing reliance on Lean for formalizing proofs and exploring mathematical concepts underscores its effectiveness and the successful adoption driven by the positive perceptions of its usefulness and ease of use.

These findings illustrate a comprehensive adoption and integration of Lean within CMU's educational framework, supported by the robust applicability of the TAM in understanding and predicting technology acceptance in higher education settings.

Impact and Implications

Firstly, the impact on teaching practices: The integration of Lean into CMU's philosophy and mathematics departments have significantly transformed teaching practices. By incorporating computer-verified proof techniques into the traditional curriculum, this shift has brought about a higher level of rigor and precision in teaching mathematical concepts. Instructors are now able to demonstrate these concepts in real-time through Lean's interactive environment, enhancing the learning experience.

Table 2

IMPACT ON TEACHING PRACTICES IN THE CMU CASE

Aspects	Interpretation
Enhanced Engagement and Interaction	The use of Lean promotes a more interactive classroom setting where students are not passive recipients of information but active participants in constructing and verifying mathematical proofs. This approach has been shown to increase student engagement and interest in complex mathematical theories by making abstract concepts more tangible and understandable.
Innovation in Pedagogy	Lean's capabilities have encouraged faculty to explore innovative pedagogical strategies, such as flipped classrooms and peer-led team learning. In these settings, students prepare by attempting proofs using Lean outside of class, then spend class time discussing difficulties or exploring more complex applications of their pre-class work. This method has not only diversified teaching approaches but also fostered a deeper understanding of content through collaborative learning.
Faculty Development	The necessity to integrate Lean into the curriculum has spurred faculty to become proficient in the tool, leading to professional development and a broader skill set among educators. This proficiency has enabled faculty members to design more dynamic and relevant course content, enhancing their teaching effectiveness and adapting to the evolving educational needs of students.

Secondly, the impact on student outcomes: The systematic use of Lean at CMU has had several positive outcomes on student learning and professional development, reflecting the effectiveness of digital tools in enhancing educational experiences.

Table 3

IMPACT ON LEARNING PRACTICES IN THE CMU CASE

Aspects	Interpretation
Improved Problem-Solving Skills	Engagement with Lean has helped students develop strong problem-solving skills, critical thinking, and logical reasoning abilities. The precision required in formalizing proofs in Lean ensures that students understand the underlying principles of mathematical arguments deeply, which is crucial for their academic and professional future.
Preparation for Professional	The skills acquired through the use of Lean are highly applicable in various technology-driven fields. Students have reported feeling more confident in their abilities to tackle complex problems and present clear, logical arguments, skills

Careers	that are highly valued in industries such as data science, software engineering, and quantitative finance.
Increased Research Opportunities	For graduate students, proficiency in Lean has opened up new research opportunities, particularly in fields requiring rigorous proof techniques and formal verification. These students are better equipped to contribute to cutting-edge research, which often involves complex computations and formalisms that can be efficiently handled using Lean.

The successful integration of Lean into CMU's philosophy and mathematics curriculum suggests a strong potential for similar digital tools to transform educational practices in other STEM fields. It also highlights the importance of supportive infrastructure, such as training programs for faculty and resource networks for students, which are critical for the adoption and effective use of technology in education.

The findings from CMU's case study illustrate the substantial benefits of incorporating advanced digital tools into higher education, both for enhancing teaching practices and improving student outcomes.

ICL Case Study

The research primarily draws upon public articles and Q&A sessions from the Xena Project's blog (Xena 2023; 2024).

Context and Implementation

At Imperial College London (ICL), teaching activities are centered around the Xena Project, which employs digital tools, specifically the Lean theorem prover, to facilitate interactive and automated theorem proving. This project is part of ICL's broader strategy to enhance digital literacy among mathematics students and to integrate computational methods into the traditional curriculum. The Xena Project is designed to make higher mathematics more accessible and engaging, particularly by using technology to bridge the gap between abstract theoretical concepts and their practical applications.

- Integration into the Curriculum: ICL has integrated the Xena Project into a variety of courses, ranging from first-year introductory courses to advanced graduate seminars. The project's implementation reflects a commitment to fostering a deep understanding of mathematics through active participation and engagement, rather than through passive observation.
- Research and Development: Beyond classroom teaching, the Xena Project also contributes to research in mathematics and computer science by providing a platform for developing and testing new algorithms for automated theorem proving. This dual focus on education and research enriches the academic environment and offers students and faculty the opportunity to explore the cutting edge of computational mathematics.

As for implementation, the specific uses of the Xena Project at ICL illustrate a robust integration of digital tools in education:

- Undergraduate Education: In foundational courses, Lean is used to introduce students to basic concepts in proof strategies and techniques. Students use Lean to construct their own proofs, which helps them visualize and understand the logical structure of mathematical arguments.

- Advanced Courses and Workshops: For more advanced students, the Xena Project offers specialized workshops where participants tackle complex mathematical problems using Lean. These workshops not only enhance students' problem-solving skills but also prepare them for using computational tools in professional research.
- Graduate Research: At the graduate level, the Xena Project is used as a research tool to formalize new mathematical theories and to verify existing ones. This rigorous application of Lean in a research context highlights its utility as a professional tool in academic mathematics.

Here are some examples of tool applications:

- Interactive Learning Modules: The Xena Project has developed interactive learning modules that allow students to explore mathematical concepts through guided discovery. These modules are integrated into the curriculum and are accessible online, allowing students to learn at their own pace outside of traditional lecture settings.
- Community Contributions: Students and faculty contribute to the Xena community by developing new libraries and modules for Lean, which are shared globally. This collaborative aspect of the Xena Project not only enhances the learning experience but also builds a sense of community among users.

The scope of Lean's use at ICL spans educational activities, from teaching fundamental concepts to conducting advanced research. This comprehensive approach not only improves the learning outcomes for students but also fosters a culture of innovation and collaboration in the mathematical sciences. Through these implementations, ICL has established a dynamic and supportive environment for exploring the potential of digital tools in mathematics education.

TAM analysis

Table 4

TAM ANALYSIS OF THE ICL CASE

Model Constructs	Analysis
External Variables	The robust community support and the availability of comprehensive resources like the Mathlib library have played a significant role in facilitating the adoption of Lean at ICL. These external variables have not only provided the necessary technical support but also created an enriching environment conducive to advanced learning and exploration of formal methods.
Perceived Usefulness (PU)	At ICL, the Xena Project has significantly enhanced students' comprehension of complex mathematical theories and proofs through the Lean theorem prover. Students recognize Lean's ability to make abstract mathematical concepts more accessible and manageable, which has greatly improved their learning outcomes. The direct application of Lean in simplifying complex proofs has

	underscored its utility in both educational and research settings, making it an invaluable resource for the students and faculty alike.
Perceived Ease of Use (PEOU)	Although initial encounters with Lean presented challenges due to its complexity, continuous usage and the structured support provided by ICL have led to a greater ease of use over time. The development of a supportive learning environment, enhanced by detailed documentation and responsive community support, has been crucial in reducing the barriers to Lean's adoption.
Attitude Toward Using (A)	The integration of Lean into the curriculum and the positive outcomes associated with its use have fostered a favorable attitude among students and faculty towards Lean. This positive reception is reflected in the enthusiasm and willingness to engage with Lean for both routine coursework and complex mathematical explorations.
Behavioral Intention to Use (BI)	The perceived effectiveness of Lean in enhancing learning outcomes, coupled with the supportive learning environment at ICL, has strongly influenced the behavioral intention to use Lean. Students and faculty are not only keen on continuing to use Lean within their current academic pursuits but are also likely to recommend its use to peers, thereby reinforcing its adoption across the academic community.
Actual System Use	Lean is extensively used across various levels of the mathematics curriculum at ICL, from introductory courses to advanced research projects. The actual usage extends beyond the classroom, with many students and faculty incorporating Lean into their research, thus indicating a deep and sustained integration of Lean into the academic practices at ICL.

These findings highlight the successful application of the TAM framework in understanding the acceptance and integration of digital tools in mathematics education at ICL. The positive results across all TAM constructs suggest that Lean is not only a beneficial tool for educational purposes but also a catalyst for advancing mathematical research and practice.

Impact and Implications

Firstly, the impact on teaching practices: The incorporation of the Xena Project and the Lean theorem prover at ICL has markedly transformed the landscape of mathematics education, infusing traditional pedagogical approaches with innovative digital tools. This transformation has brought about several significant changes in how mathematics is taught and engaged with at the institution.

Table 5

IMPACT ON TEACHING PRACTICES IN THE ICL CASE

Aspects	Interpretation
Interactive and Collaborative	Lean's introduction into the curriculum has fostered a more interactive classroom environment, where students actively engage with mathematical concepts through digital means. This interactivity has facilitated collaborative

Learning	learning, enabling students to work together on complex problems and share insights in real-time, which enhances understanding and retention of mathematical concepts.
Shift in Instructional Approaches	Faculty members have adapted their teaching strategies to leverage the capabilities of Lean, moving towards a more exploratory and student-centered approach. This shift has allowed instructors to cover more material with greater depth, as students are able to experiment and learn through direct interaction with the software, which handles the more tedious aspects of mathematical proofs.
Enhancement of Curriculum with Digital Literacy	The integration of Lean has also helped embed digital literacy into the curriculum, a crucial skill in today's technology-driven world. Students are not only learning mathematics but also how to use sophisticated computational tools that are valuable in academic and professional settings.

Secondly, the impact on student outcomes: The use of Lean in mathematics education at ICL has had profound effects on student outcomes, reflecting the tool's impact beyond mere technological integration.

Table 6

IMPACT ON LEARNING PRACTICES IN THE ICL CASE

Aspects	Interpretation
Improved Problem-Solving Skills	Students have demonstrated enhanced analytical and problem-solving skills, facilitated by the structured problem-solving environment that Lean provides. The necessity to formalize proofs and solve problems using Lean has encouraged a deeper understanding of mathematical logic and rigor.
Preparation for the Digital Environment	As students become proficient with Lean, they are better prepared for careers in sectors where mathematics and computational tools are intertwined, such as finance, data analysis, and software development. The hands-on experience with Lean equips students with a strong foundation in both theoretical and applied mathematics.
Increased Engagement and Motivation	The novel approach to learning introduced by Lean has increased student engagement and motivation. The ability to see immediate results from their input and corrections from the software provides instant feedback that is highly motivating and informative.

The successful implementation of Lean at ICL serves as a powerful model for other educational institutions looking to enhance their curriculum through digital tools. It highlights the importance of integrating technology in education not just for its own sake but as a means to improve educational outcomes and prepare students for a rapidly evolving professional landscape.

The findings from the ICL case study underline the critical role of digital tools like Lean in transforming educational practices, enhancing student learning experiences, and preparing them for future challenges in the digital society. This integration represents a forward-thinking approach to education that other institutions may look to replicate, paving the way for a broader revolution in teaching and learning in the sciences and beyond.

Comparative Analysis

A critical distinction is that the main practitioners of the CMU courses come from the philosophy department, focusing on logic and foundation of mathematics; whereas the principal practitioners at ICL are from the department of mathematics, concentrating on real-life mathematics and SOTA mathematical research.

Direct Comparison

The adoption and integration of the Lean theorem prover at CMU and ICL provide valuable insights into the effectiveness of digital tools in enhancing mathematics education. Using the TAM as a framework, a comparative analysis reveals both similarities and differences in how digital tools are perceived and utilized within these two academic settings.

Table 7

COMPARISON OF TAM CONSTRUCTS OF THE CMU AND ICL CASES

Model Construct	Comparison
External Variables	The supportive community and the availability of extensive resources played a crucial role in the successful integration of Lean at CMU. These external factors significantly influenced the positive perception of the tool's usefulness and ease of use. At ICL, the strong community support, particularly through the Xena Project, provided a rich resource environment that facilitated the adoption and effective use of Lean.
Perceived Usefulness (PU)	At CMU, Lean is viewed as highly useful for facilitating the learning and application of formal proof techniques. It is particularly appreciated for its real-world applicability in various mathematical and computational fields, enhancing students' professional preparedness. ICL reports high perceived usefulness of Lean through the Xena Project, especially for making abstract mathematical concepts more accessible and engaging. The tool's ability to visualize complex proofs significantly enhances student understanding and interest in mathematics.
Perceived Ease of Use (PEOU)	Initially, students at CMU found Lean challenging to use due to its steep learning curve. However, ongoing support and integration into the curriculum gradually improved its perceived ease of use. ICL faced similar challenges with the initial adoption of Lean. However, extensive support mechanisms and the development of tailored learning modules helped mitigate these challenges, leading to a positive shift in perception over time.

Attitude Toward Using (A)	The attitude towards using Lean at CMU is largely positive, influenced by its practical benefits and the support provided by the institution. ICL also exhibits a positive attitude toward Lean, driven by the enhanced learning experiences and the interactive nature of the tool.
Behavioral Intention to Use (BI)	The intention to continue using Lean at CMU is strong, driven by its perceived benefits in enhancing educational and professional outcomes. Similarly, at ICL, the intention to use Lean is reinforced by its positive impact on learning and research, suggesting a sustained future use.
Actual System Use	Lean is actively used across various levels of the mathematics curriculum at CMU, from introductory courses to advanced research. At ICL, Lean is similarly integrated throughout the educational spectrum, supporting both teaching and research activities.

Both CMU and ICL demonstrate strong alignments with TAM constructs, showing that despite some initial challenges with ease of use, both institutions have successfully integrated Lean into their curricula, resulting in highly positive attitudes and behavioral intentions towards its continued use. The similarities in perceived usefulness and the positive impact on student outcomes highlight the universal appeal of digital tools in enhancing mathematics education. However, the differences in initial ease of use and the strategies employed to overcome these challenges reflect the distinct educational cultures and support structures at each institution.

This comparative analysis not only underscores the adaptability of digital tools across different educational settings but also highlights the importance of institutional support and resource availability in shaping the successful integration and acceptance of technology in academia.

Factors Influencing Differences

While the overall effectiveness of the Lean theorem prover in enhancing mathematics education at both CMU and ICL is evident, there are distinct factors influencing the observed differences in how the tool is perceived and utilized at each institution. These factors include institutional policies, student demographics, and specific implementations of the technology.

Table 8

FACTORS INFLUENCING DIFFERENCES BETWEEN THE CMU AND ICL CASES

Factor	Comparison
Institutional Policies	CMU's policy of integrating computational tools into the curriculum across all levels of education creates an environment where digital tools are seen as integral to the educational process. This policy encourages early and consistent exposure to tools like Lean, which may contribute to its smoother integration and higher acceptance rates among students. ICL's approach, primarily through the Xena Project, is more focused on higher-level applications and research. The institutional policy at ICL promotes the use of digital tools primarily for complex problem solving and

	research, which may limit broader student engagement at earlier stages of education compared to CMU.
Student Demographics	The student body at CMU, with a strong representation from computer science and engineering fields, may be more accustomed and receptive to using computational tools. This familiarity likely contributes to a quicker adaptation and more positive perception of Lean's ease of use. Conversely, ICL's diverse student demographic, with a broader range of academic backgrounds, might contribute to the initial challenges in adopting Lean. Students who are less familiar with computational methods may require more time and support to adjust to using Lean effectively.
Specific Implementations of Technology	CMU's implementation of Lean includes comprehensive training sessions and integration into both core and elective courses. This broad-based approach ensures that students gain familiarity with Lean across different contexts, enhancing their comfort level and proficiency with the tool. ICL's implementation strategy, focused through the Xena Project, targets specific courses and research projects. This focused approach, while highly effective for students involved, might not provide the same level of exposure and familiarity as the broader implementation strategy seen at CMU.
Support Structures	CMU has a robust support structure that includes faculty expertise, online resources, and peer tutoring programs. This comprehensive support system is crucial in facilitating the adoption and effective use of Lean. While ICL also provides significant support through the Xena Project, the nature of this support is more specialized, primarily aimed at students and researchers directly involved in the project. This specialized support might not reach as wide an audience as the more generalized support system at CMU.
Cultural Factors	Cultural attitudes towards education and technology can also influence the adoption and use of digital tools. CMU's culture of innovation and technology-driven research may foster a more conducive environment for embracing new tools like Lean. In contrast, ICL's traditional strength in theoretical mathematics may necessitate a different approach to integrating computational tools, potentially explaining slower initial uptake but equally strong eventual support.

These factors illustrate the complexities behind the adoption and effectiveness of digital tools in educational settings. Understanding these influences can help tailor the implementation strategies to better suit the specific needs and conditions of different institutions, ultimately leading to more effective and sustainable integration of technology in education.

Reflections on TAM Model

The results from the case studies at CMU and ICL provide a robust test of the Technology Acceptance Model in a specific educational setting, particularly regarding the use of digital tools in mathematics education. Analyzing the findings from both institutions offers insights into how the TAM framework might be supported, challenged, or extended.

Support for TAM Constructs

Both CMU and ICL discovered that the Lean theorem prover was extremely useful for their educational and research needs. This aligns with the TAM's proposition that perceived usefulness plays a crucial role in the acceptance and utilization of technology. The practical applications of Lean in teaching and research settings further highlighted its value, reinforcing the notion that usefulness is a key driver of technology adoption. In both institutions, the success of Lean in fulfilling these practical needs reflects TAM's emphasis on the significant influence of perceived usefulness.

However, both institutions initially encountered challenges in using Lean, particularly in terms of its perceived ease of use. This difficulty is consistent with TAM’s view that perceived ease of use is another vital factor influencing technology acceptance. Over time, with adequate support and training, these challenges were mitigated, and perceptions of ease of use improved significantly. This evolution suggests that perceived ease of use is a dynamic construct, subject to change with appropriate interventions, a finding that supports TAM’s assertion that ease of use can be enhanced through proper guidance and experience.

Challenges to TAM Constructs

In analyzing the implementation of the Lean theorem prover, it became evident that external variables, such as institutional support, community resources, and student demographics, played a more significant role than typically highlighted in the traditional TAM. These external factors had a noticeable impact on how technology was accepted and used, suggesting that TAM may need to be expanded to give greater emphasis to these variables. A more comprehensive version of TAM could better account for the influence of external conditions, recognizing them as key drivers that shape technology adoption in educational settings.

Additionally, while TAM traditionally asserts a direct connection between behavioral intentions to use technology and actual system use, the case studies reveal that this relationship is more complex. The direct link proposed by TAM was often mediated by factors such as the availability of ongoing institutional support and the specific educational or research contexts in which the technology was implemented. This suggests that the intention to use technology does not always lead to immediate or consistent use, especially when dependent on the unique conditions surrounding the users. These findings point to the need for a more nuanced understanding of the behavioral intention and actual system use relationship within the TAM framework.

Potential Modifications and Extensions to TAM

The findings from both case studies suggest that the TAM could benefit from modifications that account for a wider range of contextual factors. Institutional policies, specific educational goals, and cultural attitudes toward technology were found to play crucial roles in shaping how users perceived and adopted technology. These factors, while typically considered external variables, could be more explicitly integrated into TAM as essential components that influence PU and

PEOU. By incorporating such contextual elements, TAM would offer a more holistic understanding of the conditions that impact technology acceptance in different environments.

Additionally, the studies reveal that technology acceptance is not static but evolves over time. As users become more familiar with the technology, their perceptions and behaviors tend to shift. This suggests that TAM could be extended to include a longitudinal perspective, allowing it to better capture these dynamic changes. A more time-sensitive approach would acknowledge that initial perceptions of technology often differ from those formed after sustained use, thereby providing a richer understanding of how technology adoption develops.

Another key finding is the role of social influence in shaping technology acceptance. Although TAM does account for subjective norms, the case studies point to the need for a broader consideration of social factors, including community engagement and peer interactions. In both settings, social influence emerged as a significant factor that shaped user behavior, suggesting that TAM could be enhanced by more explicitly recognizing the impact of community practices and social networks. This expansion would help to explain how social dynamics influence both the acceptance and sustained use of technology.

Thus, the application of TAM to the case studies at CMU and ICL not only supports many of the model's core constructs, but also highlights areas for its potential modification and extension. By integrating a more explicit focus on external variables, accommodating the dynamic nature of technology acceptance, and emphasizing the role of social and cultural influences, TAM can be made even more robust and applicable to a wider range of technological implementations in education.

Analysis Based on Actor-Network Theory

Actor-Network Theory provides a useful lens for examining the complex interplay of human actors and technological artifacts within the network of mathematics education and technology development. By applying ANT, we can expand the scope of the TAM analysis conducted in the previous sections to include a broader, more interconnected view of how digital tools like the Lean theorem prover are embedded within the academic and professional realms of mathematics.

Network of Actors and Artifacts

In understanding the network surrounding the adoption and use of Lean, it is essential to recognize the various actors and artifacts that contribute to its success. The interactions between these key players form a dynamic ecosystem where the development, application, and refinement of the tool are continuously shaped by feedback and evolving needs. This network includes a diverse range of contributors, from developers to end-users, each playing a unique role in influencing how Lean is utilized in both educational and research contexts.

- Technology Developers: These are the creators of Lean, who develop and refine the software to meet the needs of their users. They are crucial actors who initiate the network by providing the digital tool that serves as a central artifact in the network.
- Mathematicians: As users of Lean, mathematicians apply the tool in their research and problem-solving, which helps in validating and expanding the tool's capabilities. Their

feedback and innovative uses of the software further influence its development and adaptation.

- Educators in Mathematics: Often overlapping with mathematicians, these actors use Lean to teach and demonstrate complex mathematical concepts to students. Their role is pivotal in translating the capacities of the tool into educational outcomes.
- Students: The end-users of Lean in an educational setting, students interact with the tool directly. Their experiences and successes with Lean can inspire a new generation of technology developers and mathematicians, closing the loop in the network.
- Educational Institutions: These entities shape the policies and curricula that determine how and when tools like Lean are introduced to students and educators.

Interconnection with TAM Analysis

Incorporating insights from ANT into the TAM helps highlight how technology adoption is shaped by interactions within a network of actors. ANT emphasizes that factors like usefulness, ease of use, and external variables evolve through these ongoing relationships, making technology adoption a dynamic process.

- Perceived Usefulness and Ease of Use: From an ANT perspective, the usefulness and ease of use of Lean are not static qualities inherent to the tool but are outcomes of the ongoing interactions within the network. For instance, as educators and students engage with Lean, their experiences feed back into the network, potentially altering how the tool is perceived and used by others in the network.
- External Variables: ANT helps to frame external variables (such as institutional support and community resources) not merely as background factors but as active components of the network that can significantly influence the trajectory of Lean's acceptance and integration into mathematics education.
- Attitudes and Behavioral Intentions: The attitudes of educators and students toward using Lean and their intentions to use it are shaped by the dynamics within the network. For example, seeing a peer successfully use Lean can positively influence one's attitude towards it and increase their intention to use it.

Integrating ANT into the TAM framework allows for a more dynamic and systemic understanding of technology acceptance, such as theorem provers like Lean. It highlights that acceptance is not merely a product of individual cognitive assessments of usefulness and ease (as TAM suggests) but also a result of complex interactions among various human and non-human actors within a network. This integration suggests that for a technology to be successfully adopted, interventions need to consider the entire network: enhancing not only the technical aspects of the tool but also strengthening the relationships and flows of information between all actors involved.

This actor-network analysis not only complements but deepens the TAM findings by providing a holistic view of the social and material contingencies that affect the adoption and use of technological tools in educational settings. It underscores the importance of nurturing a supportive and interconnected network to foster a positive environment for technology acceptance and utilization in academia.

Key Takeaways

The comparative analysis of the TAM and ANT frameworks in the adoption of the Lean theorem prover at CMU and ICL has provided several important insights into the process of technology integration in educational settings. Both frameworks emphasize the effectiveness of Lean in enhancing mathematics education, demonstrating its broad appeal across different institutional contexts. However, the specific environment in which Lean is implemented − shaped by institutional policies, the level of engagement from educators and students, and available support systems−plays a crucial role in determining the success of its integration. Context, therefore, is key to understanding the variation in outcomes when adopting educational technologies.

The analysis also highlights the dynamic nature of technology acceptance. Rather than being a one-time decision, acceptance is an evolving process, shaped by ongoing interactions between the different actors and artifacts within the network. As educators and students engage with Lean, their perceptions of its usefulness and ease of use can shift over time, influenced by their experiences and the support they receive. This reinforces the importance of understanding technology acceptance as a fluid process that adapts as users gain familiarity with the tool.

External variables, such as institutional support, community resources, and specific educational implementations, are shown to be critical to the adoption and long-term use of Lean. These factors are not mere background elements but integral components that influence both the initial acceptance of the technology and its sustained use over time. A lack of adequate support in these areas can hinder the full integration of educational technologies, even when they have proven to be effective in other contexts.

Finally, feedback loops between users and developers play an essential role in the ongoing development of digital tools like Lean. The feedback provided by educators and students helps shape the refinement and adaptation of the software to better meet educational needs. This highlights the necessity of continuous communication between technology developers, educators, and students to ensure that the tool remains relevant and responsive to the evolving demands of academic environments.

Discussion

The integration of formal methods and theorem provers like Lean into mathematical education marks a significant transformation in teaching and learning methodologies. This discussion delves into the complexities and advantages of employing proof assistants in educational settings, proposing strategies to enhance the learning experience and examining the wider educational implications and pedagogical approaches towards teaching mathematical reasoning. By reviewing the experiences of educators who have successfully incorporated Lean into their curricula at institutions like CMU and ICL, we gain a deeper understanding of the potential and limitations of this technology.

Challenges of Incorporating Formal Methods

The introduction of theorem proving assistants like Lean in educational contexts presents several challenges:

- Initial Learning Curve: Students often encounter a steep initial learning curve, grappling with foundational mathematical concepts and the need to memorize extensive commands while navigating stringent syntax requirements. This can increase cognitive load and lead to frustration.
- Error Feedback: The complexity of interpreting error feedback from theorem provers can be daunting and demotivating, as students may find it challenging to understand and act on the feedback provided by the system.
- Technical Complexity: The requirement for a strong background in both mathematics and computer science due to the technical complexity of using theorem provers can restrict accessibility for a broader range of students.
- Integration with Curriculum: Fitting theorem provers into existing curricula necessitates substantial adjustments in teaching methods and course structures, demanding educators to thoughtfully design their curricula to meaningfully incorporate these tools.

Benefits of Using Formal Methods

Despite the initial hurdles, the adoption of formal methods and theorem provers in educational settings offers multiple benefits:

- Enhanced Understanding: The use of theorem provers enables students to explore a wide range of mathematical theories and problems. Formalizing proofs in Lean deepens their understanding of logical argument progression and abstract concepts.
- Immediate Feedback: Theorem provers provide instant feedback on proof attempts, enhancing student engagement and motivation by making the learning process dynamic and interactive.
- Skill Development: Working with Lean helps students develop essential skills in logical reasoning, problem-solving, and functional programming, which are valuable in various professional fields including software development, cryptography, and artificial intelligence.
- Collaborative Learning: The Lean community's collaborative nature fosters professional growth, offering students and mathematicians opportunities to engage in large-scale formalization projects, enhancing technical skills and building a professional network.

Strategies to Maximize Benefits and Minimize Drawbacks

Educators like Avigad have suggested several strategies to leverage the advantages of theorem provers while mitigating their drawbacks:

- Careful Task Selection: Selecting tasks that align with students' current understanding and progressively increase in complexity can help build confidence and develop skills incrementally.
- Providing Support: Offering clear guidance, tips, and strategies can demystify the learning process, making the initial steep learning curve more manageable.
- Simplifying Interaction: Developing user-friendly interfaces and tools that simplify interactions with theorem provers can make them more accessible to novices.

- Targeted Automation: Automating routine tasks within the proof process can alleviate the frustration associated with trivial steps, allowing students to concentrate on the conceptual aspects of proofs.

Broader Educational Implications and Pedagogical Approaches

The successful integration of formal methods and theorem provers at institutions like CMU and ICL has broader implications for educational practices:

- Interdisciplinary Applications: The rigorous analytical framework provided by theorem provers has applications beyond mathematics and computer science, benefiting fields such as engineering, physics, and economics.
- Enhanced Pedagogical Techniques: The immediate feedback mechanism and interactive learning environment offered by theorem provers can be replicated with other educational tools and platforms, fostering more engaging and effective learning experiences across disciplines.
- Empirical Evidence for Best Practices: The empirical approach to evaluating the effectiveness of integrating theorem provers into curricula provides valuable insights into best practices for technological integration in education. Systematic collection and analysis of data on student feedback, learning outcomes, and engagement levels can inform the development of optimized teaching methodologies.

All this comprehensive examination above highlights the promising potential of theorem provers in education, supporting a strategic and thoughtful integration that can profoundly impact teaching and learning in mathematics and beyond.

Reflections on the TAM Model

Last but not least, it is essential to offer some considerations regarding the Technology Acceptance Model itself. The integration of Lean into mathematical education at CMU and ICL offers a valuable lens through which to examine the Technology Acceptance Model and its effectiveness in educational technology settings. The case studies reveal that while TAM's core constructs of Perceived Usefulness (PU) and Perceived Ease of Use (PEOU) remain vital in predicting technology adoption, there is significant room for enhancing the model by addressing broader influences that affect technology acceptance.

Extension of TAM

The comparative analysis conducted at CMU and ICL points to the necessity of broadening TAM to more effectively capture the range of external variables and the dynamics within actor networks. Educational settings are complex systems where various factors such as institutional policies, cultural norms, and the specific educational goals profoundly impact technology adoption.

By incorporating external variables such as institutional support, community engagement, and resource availability into TAM, the model can provide a more comprehensive understanding of the factors influencing technology acceptance. Moreover, acknowledging the role of actor networks—comprising students, educators, administrators, and technology developers—can enhance the model's predictive power by illustrating how these actors interact with and influence one another in the adoption process.

Incorporation of Feedback Mechanisms

Another significant enhancement to TAM could involve the incorporation of feedback mechanisms. Feedback loops between users and developers are essential for the continuous improvement of technologies and can dramatically influence their acceptance and effectiveness in educational settings.

Feedback mechanisms reflect the dynamic nature of technology use in education, where user needs and technology capabilities evolve together over time. Incorporating these feedback loops into TAM would allow the model to not only assess initial acceptance but also the sustained use and evolution of technology as users and developers adapt to each other's needs and contributions.

For educators and policymakers, understanding these feedback mechanisms can lead to better support structures for technology integration. For developers, it highlights the importance of user input in the development cycle, encouraging more user-centered design practices that directly address the needs and challenges identified by educators and students.

Conclusion

This article has delved into the innovative integration of formal methods and theorem provers within mathematical education, with a particular emphasis on initiatives like the Xena Project and the CMU’s PAL course series, as well as other recent advances in the digitalization of mathematics. By examining the event structure, pedagogical strategies, and the utilization of theorem provers, this study has highlighted the significant potential of these technologies to enhance higher mathematical education. These tools not only facilitate a deeper understanding of complex mathematical concepts but also engage students in a more interactive and effective learning process.

The exploration has led to several potential research directions, practical implementations, and policy recommendations. Future research could investigate the scalability of integrating theorem provers across various educational levels and disciplines to examine their impact on learning outcomes and student engagement on a broader scale. Comparative studies between different theorem proving tools could further illuminate their pedagogical efficiencies, helping refine integration strategies and tool selection for optimal educational outcomes.

On a practical level, the development of a standardized framework for implementing formal methods and theorem provers in educational settings is crucial. Such a framework could include detailed guidelines for curriculum development, teacher training, and assessment methodologies tailored to leverage the unique capabilities and requirements of these technologies. This would facilitate broader adoption and ensure that educational practices are aligned with the advances in digital tools.

From a policy perspective, it is imperative that educational institutions and policymakers recognize the value of integrating advanced computational tools into curricula. Supportive policies, including funding for technology acquisition, comprehensive teacher training programs, and research into the pedagogical effectiveness of these tools, could play a pivotal role in mainstreaming these innovative approaches. Additionally, policies that encourage collaboration among educational institutions, software developers, and the industry could cultivate an ecosystem that continuously refines and enhances the use of formal methods in education.

In conclusion, the integration of formal methods and theorem provers into mathematical education represents a significant advancement in the pursuit of enhanced educational outcomes. As demonstrated by this study, the potential benefits of such integration are substantial, promising not only to deepen understanding and engagement in mathematics but also to equip students with the skills necessary to navigate and contribute to an increasingly complex technological landscape. Embracing these technologies, developing pedagogical strategies that leverage their strengths, and creating an educational environment that is both challenging and enriching for students are essential steps toward the future of educational innovation.

Acknowledgements

The authors disclose that they have no actual or perceived conflicts of interest. This research is supported by the Major Project of Key Research Base of Humanities and Social Sciences of the Ministry of Education, "Logic and Computational Research on Practical Reasoning under the Perspective of Artificial Intelligence" (Approval Number: 22JJD520001). The authors have not used artificial intelligence in the ideation, design, or write-up of this research as per Crawford et al. (2023). The authors list the following CRediT contributions: [Zhenyu Sun: Data gathering, Investigation, Data curation, Original draft preparation; Ruixue Yuan: Conceptualization, Methodology, Reviewing and Editing; Xuezhi Liu: Supervision.]

References

Antonietti, C., Cattaneo, A., & Amenduni, F. (2022). Can teachers' digital competence influence technology acceptance in vocational education? *Computers in Human Behavior, 132*, 107266. https://doi.org/10.1016/j.chb.2022.107266

Anuratha, K. (2020). Promoting learning outcomes using digital literacy. *International Journal of Advance Research, Ideas and Innovations in Technology, 6*(2), 318-323. https://www.ijariit.com/manuscript/promoting-learning-outcomes-using-digital-literacy/

Avigad, J. (2021a). *Teaching logic and mechanized reasoning with Lean 4* [Beamer slides]. Carnegie Mellon University. https://www.andrew.cmu.edu/user/avigad/Talks/fmtea.pdf

Avigad, J. (2021b). *Lean Together 2021: Teaching with proof assistants* [Beamer slides]. Carnegie Mellon University. https://www.andrew.cmu.edu/user/avigad/Talks/education.pdf

Avigad, J. (2023). *Teaching undergraduate mathematicians and computer scientists how to formalize mathematics* [Beamer slides]. Carnegie Mellon University. https://www.andrew.cmu.edu/user/avigad/Talks/loughborough2.pdf

Avigad, J. (2024). *Proof assistants and mathematics education* [Beamer slides]. Carnegie Mellon University. https://www.andrew.cmu.edu/user/avigad/Talks/cbms.pdf

Awodey, S. (2014). Structuralism, invariance, and univalence. *Philosophia Mathematica, 22*(1), 1-11. https://doi.org/10.1093/philmat/nkt030

Bezem, M., Buchholtz, U., Cagne, P., Dundas, B. I., & Grayson, D. R. (2022). *Symmetry*. [Book in progress]. https://unimath.github.io/SymmetryBook/book.pdf

Buzzard, K., Commelin, J., & Massot, P. (2020). Formalising perfectoid spaces. *Proceedings of the 9th ACM SIGPLAN International Conference on Certified Programs and Proofs (CPP 2020)*, 299-312. https://arxiv.org/abs/1910.12320

Buzzard, K. (2022a, July 6). *The rise of formalism in mathematics* [Plenary speech]. International Congress of Mathematicians (ICM) 2022. YouTube. https://www.youtube.com/watch?v=SEID4XYFN7o

Buzzard, K. (2022b). *Formalising mathematics* [Course notes]. Imperial College London. https://www.ma.imperial.ac.uk/~buzzard/xena/formalising-mathematics-2022/

Davis, F. (1989). Perceived usefulness, perceived ease of use, and acceptance of information technology. *MIS Quarterly, 13*(3), 340-391. https://doi.org/10.2307/249008

Davies, A., Veličković, P., & Others. (2021). Advancing mathematics by guiding human intuition with AI. *Nature, 600*(7887), 70-74. https://doi.org/10.1038/s41586-021-04086-x

Fawzi, A., Balog, M., & Others. (2022). Discovering faster matrix multiplication algorithms with reinforcement learning. *Nature, 610*(7930), 47-53. https://doi.org/10.1038/s41586-022-05172-4

Gowers, W. T., Green, B., Manners, F. & Tao, T. (2023). On a conjecture of Marton. *arXiv*. https://arxiv.org/abs/2311.05762

Gukov, S., Halverson, J., Manolescu, C., & Ruehle, F. (2023). Searching for ribbons with machine learning. *arXiv*. https://arxiv.org/abs/2304.09304

Ince-Muslu, B., & Erduran, A. (2021). A suggestion of a framework: Conceptualization of the factors that affect technology integration in mathematics education. *International Electronic Journal of Mathematics Education, 16*(1), em0617. https://doi.org/10.29333/iejme/9292

Latour, B. (2005). *Reassembling the social: An introduction to actor-network-theory*. Oxford University Press.

Lean. (2023). *Programming language and theorem prover - Lean* [Software]. https://lean-lang.org/

Li, M. (2024). Assessing Chinese primary mathematics teachers' self-efficacy for technology integration: Development and validation of a multifaceted scale. *Asian Journal for*

Mathematics Education, 0(0), 27527263241254496. https://doi.org/10.1177/27527263241254496

Noh, N., Raju, R., Eri, Z., & Ishak, S. (2021). Extending technology acceptance model (TAM) to measure the students' acceptance of using digital tools during open and distance learning (ODL). *IOP Conference Series: Materials Science and Engineering, 1176*, 012037. https://doi.org/10.1088/1757-899X/1176/1/012037

O'Dea, X., & O'Dea, M. (2023). Is artificial intelligence really the next big thing in learning and teaching in higher education? A conceptual paper. *Journal of University Teaching & Learning Practice, 20*(5), 05. https://doi.org/10.53761/1.20.5.05

PAL. (2023). *The Pure and Applied Logic Program*. Carnegie Mellon University. https://logic.cmu.edu/

Polymath. (2023). *Polymath Project*. https://polymathprojects.org/

The Univalent Foundations Program. (2013). *Homotopy type theory: Univalent foundations of mathematics.* https://homotopytypetheory.org/book/

Scholze, P. (2022). Liquid tensor experiment. *Experimental Mathematics, 31*(2), 349-354. https://doi.org/10.1080/10586458.2021.1926016

Sinclair, N., & Yerushalmy, M. (2016). Digital technology in mathematics teaching and learning. In *The second handbook of research on the psychology of mathematics education* (pp. 235-274). Springer. https://doi.org/10.1007/978-94-6300-561-6

Soydaş, E. (2023). *Factors affecting teachers' technology acceptance and usage for teaching mathematics* [Master's thesis, Middle East Technical University]. https://open.metu.edu.tr/bitstream/handle/11511/107741/10604489.pdf

Teeroovengadum, V., Heeraman, N., & Jugurnath, B. (2017). Examining the antecedents of ICT adoption in education using an extended technology acceptance model (TAM). *International Journal of Education and Development using Information and Communication Technology (IJEDICT), 13*(3), 4-23. https://files.eric.ed.gov/fulltext/EJ1166522.pdf

Trinh, T. H., Wu, Y., & Others. (2024). Solving olympiad geometry without human demonstrations. *Nature, 620*(7995), 476-482. https://doi.org/10.1038/s41586-023-06747-5

Voevodsky, V. (2006). *A very short note on homotopy lambda-calculus* [Mailing list post]. https://www.math.ias.edu/~vladimir/Site3/Univalent_Foundations_files/Hlambda_short_current.pdf

Wagner, A. (2023). Finding counterexamples via reinforcement learning. In *IPAM Machine Assisted Proofs Workshop*. https://users.wpi.edu/~zadam/Summer_school_Hausdorff_Day_1__RL

Wolfram, C. (2010). Teaching kids real math with computers. *TED Talks*. https://www.ted.com/talks/conrad_wolfram_teaching_kids_real_math_with_computers

Xena. (2023). *The Xena Project*. https://www.ma.imperial.ac.uk/~buzzard/xena/

Xena. (2024). *Xena: Mathematicians learning Lean by doing*. Imperial College London. https://xenaproject.wordpress.com/

Yeo, S., Rutherford, T., & Campbell, T. (2022). Understanding elementary mathematics teachers' intention to use a digital game through the technology acceptance model. *Education and Information Technologies, 27*(8), 11515–11536. https://doi.org/10.1007/s10639-022-11073-w

Zogheib, B., Rabaa'i, A., Zogheib, S., & Elsaheli, A. (2015). University student perceptions of technology use in mathematics learning. *Journal of Information Technology Education: Research, 14*, 401-422. https://doi.org/10.28945/2315