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Mathematical Argumentation in Higher Education: A Systematic Literature Review

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Abstract

Interest in research on mathematical argumentation is increasing, leading to growth in the field. However, empirical research on mathematical argumentation in higher education presents challenges in organising a comprehensive overview due to its diversity and variety of underlying themes and tasks. Therefore, the aim of this study is to provide a systematic review of studies that focus on mathematical argumentation in higher education. We conducted a systematic literature review with the guiding questions: "What are the characteristics of tasks used for mathematical argumentation?" and "What themes are being explored in studies on mathematical argumentation in higher education?" The results of this review indicate a strong correlation among mathematical argumentation in higher education, mathematical proof tasks, and instructional design. Additionally, empirical studies conducted between 1995 and 2023 have identified four primary research themes encompassing student abilities, collaboration, designing and teaching, and assessment. Our analysis offers a detailed examination and appraisal of each of these themes and associated tasks, in conjunction with current trends, facilitating a more organized understanding of this multifaceted research discipline.

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Introduction

In recent years, mathematics curricula worldwide have shifted their focus to developing students' ability to construct arguments for and against mathematical claims and to generate or investigate mathematical conjectures (Kollar et al., 2014; Nama & Ayalon, 2023). Constructing mathematical arguments and critiquing the reasoning of others are considered important goals of mathematics education in school curricula. These skills promote the development of new perspectives and further study (CCSSI, 2018; NCTM, 2000). In secondary and post-secondary education, constructing arguments is a well-researched and significant objective (Schwaighofer et al., 2015). This is because argumentation is recognized as a key element in teaching and learning mathematics (Cervantes-Barraza & Cabañas-Sánchez, 2022; Erkek & Isiksal-Bostan, 2019). Moreover, argumentation plays a crucial role in creating equitable learning opportunities in the classroom by shaping classroom discourse, which, in turn, influences students' understanding of mathematics (Francisco, 2022).

The role of argumentation in the development of mathematical thinking and the learning and teaching of mathematical concepts has been emphasized in numerous studies (Demiray et al., 2021). The examination of argumentation provides insight into students' mathematical thinking and offers a platform for the emergence, discussion, and eventual consolidation of mathematical ideas (Conner et al., 2022). Research indicates that argumentation in mathematics education involves investigating, challenging, and evaluating a range of positions, as well as supporting, objecting to, and justifying diverse ideas and hypotheses. This not only facilitates a deeper understanding of the subject, but also promotes critical thinking skills (Asterhan & Schwarz, 2016; Weber et al., 2008). As such, argumentation is considered central to both mathematics education and research (Mariotti et al., 2018).

Research has been conducted in various scientific journals on argumentation in mathematics education, using a range of methods (Kartika et al., 2023). An increasing body of research concentrates on how students, mathematics teachers, and mathematicians engage in argumentation while working with mathematical concepts (Inglis et al., 2007). In addition, academic articles provide valuable information and data for researchers. For instance, Campbell et al. (2019) utilized a systematic review methodology to examine research studies on proof and argumentation in mathematics for K-12 students. Nevertheless, the use of these systematic reviews for mathematical argumentation in higher education has yet to be published. This study concentrates on analytical methods for systematic reviews to identify studies on mathematical argumentation in higher education published in scientific journals. More specifically, this paper aims to address two research questions regarding mathematical argumentation (MA) in higher education: (1) what are the characteristics of tasks used for MA? and (2) what themes are being explored in studies on MA in higher education? Conducting a systematic literature review (SLR) will contribute to the current understanding of MA in higher education and the extent of investigation in each research focus.

Literature

Argumentation has multiple definitions that vary depending on the specific discipline and context (Demiray et al., 2023). Additionally, it can arise through different methods including questioning, persuasion, negotiation, or disagreement (agree or disagree) (Kartika & Budiarto, 2022). Argumentation is manifested by a process "...that takes into account all of the assumptions (initial data and warrants) of the entire argumentation, but which hides the relationships between these assumptions." (Knipping & Reid, 2015, p. 90). Toulmin (2003) clarified argument structure by describing three interrelated core components of an argument: conclusion, data, and warrant. The conclusion is the claim, a statement that is being argued for (Zambak & Magiera, 2020). During the argumentation process, data that support the claim are produced. It is crucial to provide validation for the association between the data and the claim. This statement serves as the foundation and can be expressed through mathematical principles, rules, axioms, definitions, or theorems (Urhan & Zengin, 2023). According to Cardetti and LeMay (2019), a mathematical argument can be defined as a series of statements and corresponding reasons (data, warrants, backing) that strive to prove the validity or falsehood of a claim (conclusion).

A claim is an objective statement that is accepted within the context of the argument being presented. Data support these claims, while warrants serve as a means of connecting the data to the claims, providing additional information to strengthen the data and reduce ambiguity (Conner et al., 2014). Conclusions propose claims with data serving as evidence to substantiate these claims. A warrant is a factual or categorical statement that pertains to both the conclusion and the explanation of the data (Freeman, 2011). Its purpose is to connect the data to the conclusion, demonstrating the conclusion's validity and explaining how the data supports the claims. Moreover, Toulmin (2003) describes three additional argument components, namely: modal qualifier, backing, and rebuttal, which may not be present in all arguments. The modal qualifier expresses the level of certainty, the backing serves as supplementary support for the warrant, and the rebuttal either rejects the warrant or provides support for a counter-argument.

In a mathematical classroom setting, arguments are warranted based on objective criteria. These criteria include mathematical knowledge, verification, authority, external validation, interpretation, patterns, method, calculation, visual representations, informal mathematical knowledge, and provided information (Conner, 2012). Mathematical argumentation in this study involves constructing compelling arguments to demonstrate the validity of mathematical statements or solutions to mathematical problems.

Method

This study employed a methodological approach centred on performing a systematic review of the scholarly literature. A systematic review of the literature is a challenging academic endeavour distinguished by a precisely defined and rigorous framework of principles (Oakley, 2012). These principles emphasise the review's thoroughness, impartiality, and justifiable methodology and implementation, emphasising transparency and responsibility (Dixon-Woods, 2011). We conducted our systematic literature review following the methodological framework outlined by Gough et al. (2013). This approach, which originated from political studies (Joklitschke et al., 2021), has been modified for use in the field of mathematics education by both Nilsson et al.

(2018) and Joklitschke et al. (2021). The following section outlines the ten steps involved in this methodology.

Steps 1 and 2: Need and Review Question

The reasons for conducting our extensive review and related research investigation were outlined in the introductory section of the scholarly manuscript.

Step 3: Scope

To assess article quality in our review, we exclusively chose peer-reviewed journal articles, and excluded non-journal articles such as book chapters, theses, and conference proceedings. Book chapters were excluded from the review because of their relatively narrow focus and potential lack of comprehensive coverage compared to full-length research articles. While book chapters can provide valuable insights, they often present specific case studies or discussions within a limited context. We restricted our selection further to articles published in English. This approach facilitates the full reproducibility of our analysis. Our main focus was on empirical articles published between 1995 and December 3, 2023. We aimed to present empirical research on mathematical argumentation. The search year was determined by Gotz Krummheuer's initial scientific article on mathematical argumentation literature, published in 1995. Our study specifically targeted university students, while excluding those on K-12 levels, in-service teachers, and teachers (Table 1).

Table 1

The Criteria of Inclusion and Exclusion

Inclusion	Exclusion
Peer-reviewed journal articles	Book chapters, theses, conference proceedings or other non-journal articles
Articles published in English	Non-English articles
Empirical studies in mathematics learning or mathematics education	Studies on other fields or literature reviews
Articles published from 1995 to 2023	Articles published outside of the specified time frame
Focus on mathematical argumentation in higher education	Others (for example, mathematical argumentation in K-12 levels or mathematical argumentation in teacher or in-service teachers)

Step 4: Search

To enhance the identification of pertinent research papers exploring mathematical argumentation in higher education, we conducted a thorough search utilising the Publish or Perish software. Our search involved the implementation of search strings with asterisks and Boolean operators, which allowed us to extract specific terms from article titles, abstracts, and keywords. We decided to include articles with the words argument* and math* in the title and to include articles with the words "university" or "higher education" or "tertiary" or "college" or "graduate" or "undergraduate" or "postgraduate" or "teacher education" or "pre-service teacher" or "prospective teacher" in the abstract or keywords. The rationale behind this methodology is to include articles that focus on the concept of argumentation. An excerpt of the search strategy for the search terms is presented in Table 2.

Table 2

Search Strings

Databases	Search Items
- Scopus - Google Scholar	(argument* AND math*) AND ("university" OR "higher education" OR "tertiary" OR "college" OR "graduate" OR "undergraduate" OR "postgraduate" OR "teacher education" OR "pre-service teacher" OR "prospective teacher")

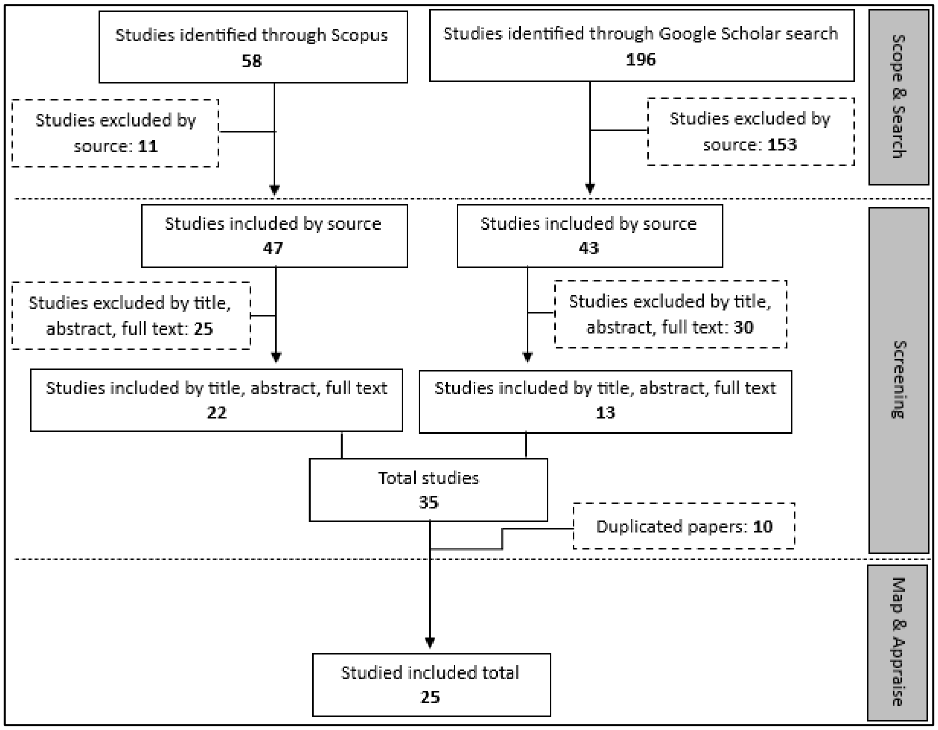
The Scopus and Google Scholar databases were selected as potential sources to obtain articles related to the established objectives. The Scopus database was selected due to its extensive coverage, surpassing other databases and incorporating around 70% more sources than the Web of Science (WoS) (Mongeon & Paul-Hus, 2016). Google Scholar is an attractive resource for researchers because of its free accessibility, apparent indexing of a significant volume of academic articles, ability to facilitate the export of individual citations, and its provision of citation tracking features (Haddaway et al., 2015). The keyword search was restricted to the existence of the search term within the titles, abstracts, and keywords of the articles. Advanced search options were used to enter search terms and apply syntactical matching with search engine operators. As a result, we retrieved 58 documents from the Scopus database and 196 documents from the Google Scholar database. To increase the precision of our search in Google Scholar and to identify publications of higher quality and relevance to the field of mathematics education, we specifically included articles published in journals listed in the Scopus database under the category of education. In total, 43 relevant results were found. Additionally, out of the 58 articles obtained from the Scopus database, only 47 are articles published in journals. The remaining 11 articles that were excluded consist of 6 conference proceedings, 2 book chapters, 2 errata and 1 article sourced from the journal Mathematical Medicine and Biology. These data were extracted by the first author, and then verified by the second, third and fourth authors. In addition, we discussed via email and online meetings to ensure the relevance of the articles based on the inclusion criteria.

Step 5: Screening

We screened a total of 90 articles (47 from Scopus and 43 from Google Scholar) by reviewing titles, abstracts, keywords and full text. Articles that did not address mathematical argumentation in higher education or that did not meet the criteria 2-5 from step 3 were excluded. The screening resulted in the inclusion of 25 articles, as shown in Figure 1, for further analysis. The reduction to only 27.8% of the original dataset - specifically 25 out of 47 + 43 articles - is a significant achievement. The exclusions can be primarily attributed to theoretical review articles, duplication, and the inclusion of teachers as research subjects. These 25 articles were distributed across 20 different journals, with Cognition and Instruction, Educational Studies in Mathematics, International Journal of Computer-Supported Collaborative Learning, International Journal of Science and Mathematics Education, and Journal of Mathematics Teacher Education having the greatest number of articles.

Figure 1

PRISMA Diagram



Step 6: Code

As part of our research objective, we analysed various sections of primary articles - from introductions to conclusions and recommendations. Our aim was to generate a review table that identified key themes, methodologies, findings, and recommendations. We also noted that an article can relate to multiple thematic categories. The diversity of emphasis in studies on mathematical argumentation was guided by these themes. The findings of the primary articles hold the potential to improve our understanding of mathematical argumentation. The recommendations also offer insights for future research in this domain.

Step 7 and 8: Map and Appraise

The mapping phase is essential to conduct a content analysis of the coded thematic categories. These themes were then subjected to inductive clustering, with a focus on mathematical argumentation concepts, using the VosViewer software. During the appraisal phase, an evaluation was conducted to examine various mathematical argumentation concepts in higher education. The main goal was to examine the theoretical foundations and practical usage of the subject. The theoretical framework was further developed by utilizing theories from not only the primary articles but also from two books (Aberdein & Dove, 2013; Mirza & Perret-Clermont, 2009).

Steps 9 and 10: Synthesize and Communicate

The final outcome of the literature review process that consolidates the findings and addresses the research question is represented by the synthesis phase (Joklitschke et al., 2021).

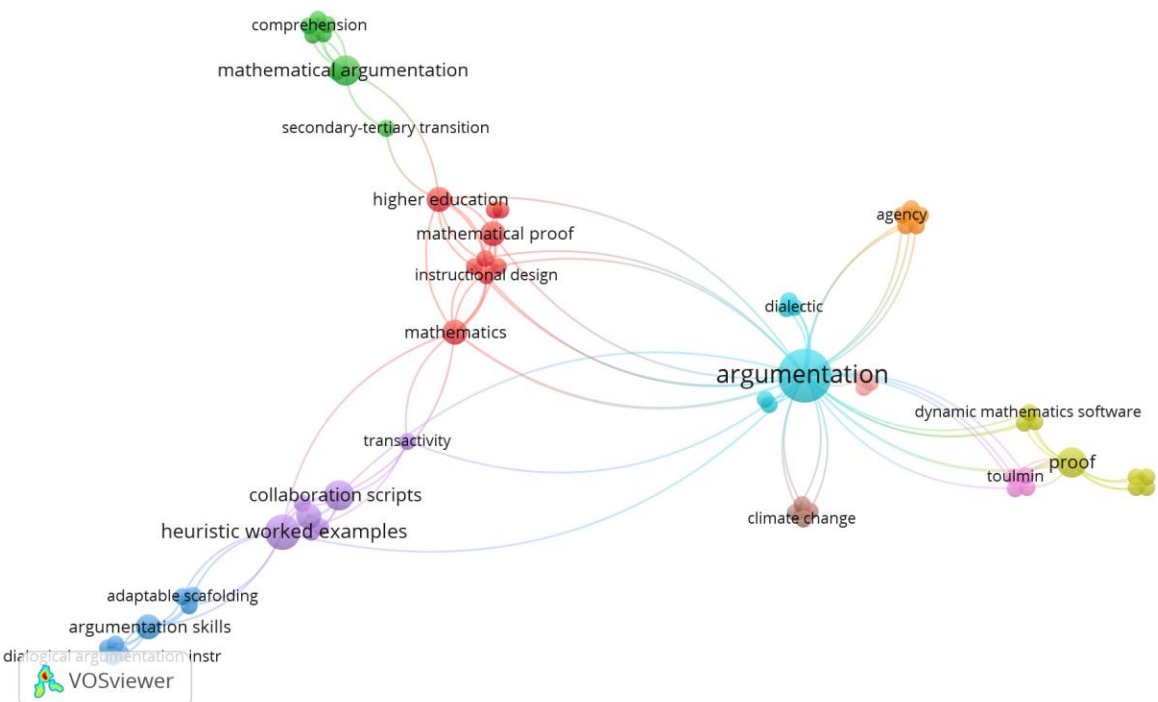
Dissemination of the obtained results occurs during the communication stage (Gough et al., 2013). The results and discussion sections will elaborate on both the synthesis and communication aspects.

Results and Discussion

This section provides a thorough analysis of the mathematical argumentation patterns observed in higher education studies. It outlines the key characteristics of the primary articles and presents a detailed discussion addressing research questions. Using VosViewer, a visualization software, 25 titles obtained through the title search are included to illustrate the overall trends within these studies. The visual representation of the study clusters, derived from all keywords (abstracts and keywords) of the 25 articles, is presented in Figure 2.

Figure 2

Clusters of Studies Analysing Mathematical Argumentation



There are ten clusters on the map, each shaded with a distinct colour gradient. Keywords with strong correlations are grouped using similar colours, while the links between them indicate instances when these keywords appear together in publications. The size of the keyword icon reflects the frequency of its appearance in these publications (van Eck & Waltman, 2022). The clusters of keywords are shown in Table 3.

Table 3

Keyword Clustering in the Study of Mathematical Argumentation

Cluster	List of Keywords
1	higher education, instructional design, mathematical proof, mathematics, mathematics education, pre-service middle school mathematics teachers, proof methods, resource-based cognitive skill, whole-task learning.
2	adversity quotient, comprehension, course work, interest, mathematical argumentation, proof scheme, secondary-tertiary transition, text-domain.
3	adaptable scaffolding, argumentation skill, CSCL script, dialogical argumentation instructional model, maths-in-physics problems, mechanics, pre-service teachers, self-regulation.
4	dynamic mathematics software, Habermas's construct of rationality, infusion learning, learning strategy, mathematical argumentation skill, pre-service teacher, proof, Toulmin's model.
5	collaboration scripts, general prior achievement, heuristic worked examples, mathematical argumentation skills, synergistic scaffolding, transactivity, and working memory capacity.
6	argumentation, dialectic, GeoGebra, logic, rhetoric, prospective middle school mathematics teacher, teacher knowledge.
7	agency, analogical reasoning, collaboration, mathematical referents, and semantic warrant.
8	climate change, critical thinking, initial teacher education, socio-scientific issues.
9	informal logic, number theory, reasoning, Toulmin.
10	didactic suitability criteria, master's degree final project, mathematical modelling.

It showed that in cluster 1, mathematical argumentation in higher education was closely related to mathematical proof and instructional design. Understandably, mathematical argumentation and proficiency in proof can be regarded as cognitive abilities based on accessible resources, a shift that often occurs when advancing through higher education (Renninger et al., 2023; Sommerhoff et al., 2021). Furthermore, the standard arrangement of mathematical lectures at the university level highlights the importance of proofs and proving (Cardetti & LeMay, 2018; Nagel et al., 2018). For example, Bleiler et al. (2013) developed educational interventions after reviewing literature that suggested proof validation presents challenges for undergraduate students.

The Scope, Designs, and Activities of Studies

The majority of the selected articles were published between 2019 and 2021 (32%, n=8), and focused primarily on undergraduate students (40%, n=10). Most studies used quantitative research designs (52%, n=13). These studies were conducted in several countries, including the United States (6 articles), Germany (5), the United Kingdom (4), Turkey (4), Indonesia (2), Spain (2), South Africa (1), and Colombia (1) (see Appendix A).

Furthermore, some studies involved the design and implementation of a series of activities, including task-based interviews (Inglis et al., 2007; Marchant et al., 2021; Morris, 2007; Simsek, 2021), collaborative work among students (Walter & Barros, 2011), validation of skills in mathematical arguments (Bleiler et al., 2013; Castro et al., 2021), open-ended questions illustrating the thinking process and the development of students' argumentation (Nagel et al., 2018), argumentative tasks (Erkek & Bostan, 2019), explaining geometry tasks (Simsek, 2021), the use of a common socio-scientific scenario, climate change (Ariza et al., 2021), a submodule on mathematical modelling (Ledezma, et al., 2022), and collaborative argumentation and proving processes using dynamic mathematics software GeoGebra (Urhan & Zengin, 2023).

In contrast, some studies conducted experiments to directly compare the persuasiveness of various types of mathematical arguments in two to four treatment sessions (Hidayat et al., 2018; Inglis & Mejia-Ramos, 2009a; Inglis & Mejia-Ramos, 2009b; Iwuanyanwu & Ogunniyi, 2020; Kollar et al., 2014; Schwaighofer et al., 2017; Sommerhoff et al., 2021; Tristanti & Nusantara, 2022; Vogel et al., 2022; Zambak & Magiera, 2020;). On the other hand, Demiray and Bostan (2017) used a cross-sectional survey design to examine the competence of Turkish pre-service middle school math teachers in conducting valid proofs for statements about numbers and algebra. Renninger et al. (2023) employed cluster analysis using the Assessment of Mathematical Comprehension (AMC) to examine participants' interest in mathematics and text domain, as well as their comprehension of mathematical argumentation.

Definition and task characteristics of Mathematical Argumentation in Higher Education

The study of mathematical argumentation in higher education explored two primary subjects: the definition and characteristics of tasks related to mathematical argumentation. It is crucial to examine these subjects first in a broader context before narrowing the focus to mathematical argumentation. Our investigation into the definition of mathematical argumentation has uncovered two main perspectives: one that views it as a skill and the other as an activity. These viewpoints are presented in two books and 25 principal articles. In terms of task characteristics, we have identified 14 key attributes inherent to these tasks: (1) proof construction tasks, (2) evaluating conditional statements, (3) explaining geometry tasks, (4) collaborative scripts and heuristic worked examples, (5) solving crypto-arithmetic problems, (6) determining the derivative of a function's graph, (7) mathematical conjecturing problem in elementary number theory, (8) designing modelling tasks, (9) limit involving $sin\, x/x$, (10) a statement of Young's Inequality, (11) conceptual maths-in-physics (MIP) problems in mechanics, (12) the use of a common socio-scientific scenario, climate change, (13) designing and implementing the instruction, and (14) math images Wiki page about Fibonacci numbers.

Definition of Mathematical Argumentation in Higher Education

Numerous academic sources, including Inglis and Mejia-Ramos (2009), have defined mathematical argumentation in higher education as the ability to generate novel arguments, articulate existing ones, and comprehend the provided arguments. This skill involves evaluating or comprehending the arguments through the application of valid rules of inference, axioms, definitions, and established conclusions. University students should identify logical sequences of deductive arguments that support hypotheses in a theoretical framework. Additionally, they must effectively communicate, explain, and persuade others of the validity of their mathematical reasoning and conclusions by established mathematical criteria (Vogel et al., 2022; Walter & Barros, 2011).

These definitions align with the perspective of Andriessen (2009), who suggests that during argumentative activities, university students try to persuade each other by presenting compelling arguments in support of their positions and by challenging the arguments put forth by their peers. In mathematics, the process of solving a problem and following a clear method of reasoning to reach a solution inherently exhibits argumentative qualities (Banegas, 2013). Andriessen (2009) points out that arguing is particularly valuable for students seeking to comprehend open problems and areas of uncertainty, ultimately enhancing their understanding of scientific domains. Therefore, mathematical argumentation in higher education is both an activity and a skill that serves to assess the validity of arguments, prove conjectures, and engage in the problem-solving process. This activity involves constructing deductive arguments using valid rules of inference, axioms, definitions, and previously established conclusions to demonstrate the validity of mathematical statements or solutions to mathematical problems.

Task Characteristics

Proof construction tasks were the most frequently observed, occurring five times and constituting 35.7% of the total tasks examined. For example, Nagel et al. (2018) created three geometry proof tasks for first-year university students in mathematics to illustrate the thinking process and development of students' argumentations. Students were asked to prove three theorems: (1) why the three bisectors of a triangle intersect, (2) why the Thales theorem is true, and (3) the Pythagorean theorem. Another example of a task was proof construction tasks based on real analysis lectures (Sommerhoff et al., 2021) and proof problems in the pyramid (Tristanti & Nusantara, 2022). In addition, the categories that included evaluating conditional statements, evaluating students' arguments and explaining geometry tasks to both peers and students in the classroom had a total of four instances, accounting for 28.6% of the tasks. Collaborative scripts, and heuristic worked examples had a total of two instances. Categories such as solving crypto-arithmetic problems, determining the derivative of the graph of a function, mathematical conjecturing problems in elementary number theory, designing modelling tasks, limit involving $sin\, x/x$, a statement of Young's Inequality, conceptual maths-in-physics (MIP) problems in mechanics, the use of a common socio-scientific scenario, climate change, designed and implemented the instruction, and the math images Wiki page about Fibonacci numbers each had one instance, representing 7.1% of the tasks.

Themes of studies

An analysis was carried out to identify the themes and the research design of the articles that were selected. The methods and findings of the 25 studies were examined by the first author and then verified by the second, third and fourth authors to develop appropriate themes, with common themes also considered. The process involved reading the research objectives, methodology, and procedures to identify themes and commonalities across studies. In this process, four themes were identified: Students' abilities, collaboration, designing and teaching, and assessment. Table 4 presents the main themes of the articles.

Table 4

Key Themes of the Main Articles

No.	Study	Theme Found Students' Abilities	Collaboration	Designing and Teaching	Assessment
1.	(Morris, 2007)	√			
2.	(Inglis & Mejia-Ramos, 2007)	√			
3.	(Inglis &Mejia-Ramos, 2009a)	√			
4.	(Inglis &Mejia-Ramos, 2009b)	√			
5.	(Walter & Barros, 2011)		√		
6.	(Bleiler et al., 2013)	√			
7.	(Kollar et al., 2014)		√		
8.	(Vogel et al., 2016)	√			
9.	(Schwaighofer et al., 2017)		√		
10.	(Demiray & Bostan, 2017)	√			
11.	(Hidayat et al., 2018)	√			
12.	(Nagel et al., 2018)	√			
13.	(Erkek & Bostan, 2019)	√			
14.	(Zambak & Magiera, 2020)	√			
15.	(Iwuanyanwu & Ogunniyi, 2020)	√			
16.	(Castro et al., 2021)			√	
17.	(Simsek, 2021)	√			
18.	(Sommerhoff et al., 2021)	√			
19.	(Ariza et al., 2021)	√			
20.	(Marchant et al., 2021)	√			
21.	(Vogel et al., 2022)	√			
22.	(Tristanti & Nusantara,2022)	√			
23.	(Ledezma et al., 2022)			√	
24.	(Renninger et al., 2023)				√
25.	(Urhan & Zengin, 2023)		√		

The 'Students' Abilities' theme focuses on studies that identify the diverse understandings, skills, and beliefs that impact students' assessments of mathematical arguments in classroom settings. Meanwhile, the 'Collaboration' theme includes research on students' collaborative argumentation in developing mathematical problem-solving strategies, often without prior instruction on solution methods. The 'Designing and Teaching' theme pertains to studies that analyse students' explanations and reflections on the design of mathematical tasks and the implementation of teaching and learning sequences during their educational internships. Studies in ‘Assessment’ explored students’ comprehension of mathematical argumentation concerning specific text and learner characteristics. Each issue and future recommendations were outlined.

Students’ Abilities during Mathematical Argumentation

The main theme of the discussed articles is student ability, with 18 of them aiming at exploring this topic. Morris (2007), for example, conducted a study on transcriptions of third-grade lessons to investigate if pre-service teachers rated students' arguments consistently when asked to demonstrate generalization across distinct instructional settings. The study aimed to ascertain whether pre-service teachers could consistently assess student arguments in various educational contexts. The study investigated pre-service teachers' assessments of students' responses under two experimental conditions. In the first scenario, a student presented a sound argument that validated the generalisation, whereas in the second scenario, this student's response was not included in the transcript. In another investigation by Inglis et al. (2007), numerous task-based interviews were conducted with highly accomplished mathematics graduates pursuing postgraduate degrees. The interviews aimed to explore successful mathematicians' evaluation of conditional statements. In a study by Inglis and Mejia-Ramos (2009), three experiments were conducted. These experiments demonstrated that in specific scenarios, mathematics students and researchers exhibit higher levels of persuasion towards mathematical arguments associated with an authority figure compared to those that are not.

Bleiler et al. (2013) collaborated with aspiring secondary mathematics educators to study twelve mathematical arguments produced by high school students. The prospective teachers assessed each item numerically and provided brief written comments delineating the arguments' attributes and limitations. They devised and administered a set of five activities that targeted pre-service mathematics teachers’ (PSMTs') proficiency in recognizing and validating mathematical arguments, with a specific focus on inductive/deductive reasoning and local/global components of mathematical arguments. Hidayat et al. (2018) conducted a study to investigate the impact of pre-service mathematics teachers' adversity quotient on their performance in validating mathematical arguments. Nagel et al. (2018) devised a set of three objective open-ended questions to examine the evidence-based reasoning and argumentative skills' growth among students.

Zambak and Magiera (2020) conducted a study on a teaching experiment with pre-service teachers (PSTs) to enhance their ability to create and evaluate mathematical arguments. The experiment aimed to improve flexibility in using various problem-solving strategies and aid understanding of mathematical argumentation. The course activities aimed to attain three primary goals: (a) to enhance the ability of PSTs to formulate mathematical arguments, (b) to improve their ability to assess and appraise the quality of mathematical arguments, and (c) to reinforce their comprehension of problems and classroom situations that encourage students to reason and create mathematical arguments.

Sommerhoff et al. (2021) applied two instructional techniques when students worked on proof construction tasks: (i) a sequential strategy that sequentially emphasised and supported each resource of mathematical reasoning and proof skills, and (ii) a concurrent strategy that simultaneously emphasised and supported multiple resources. Vogel et al. (2022) carried out two experiments. In the first experiment, researchers conducted a comparison of the effects of adaptable and non-adaptable computer-supported collaborative learning (CSCL) scripts on the learning of mathematical argumentation skills (MAS). In the second experiment, they compared the effects of adaptive and non-adaptive heuristic work examples on the learning of MAS. Finally, the aim of Tristanti and Nusantara's (2022) study was to assess the impact of the infusion learning strategy on the mathematical argumentation skills of students, with a particular focus on pre-service mathematics teachers.

In summary, the studies on students' abilities in mathematical argumentation have predominantly been conducted in controlled or designed environments. These studies have explored various instructional approaches aimed at enhancing these skills. Consequently, generalising these findings to contexts beyond undergraduate mathematics should be approached with caution, and it necessitates a broader range of studies across various settings for validation (Vogel et al., 2022). It is important to acknowledge that their focus on students' written responses may have limitations in assessing students' reasoning skills, as written responses might not always accurately represent their cognitive processes (Zambak & Magiera, 2020). Future research should consider incorporating alternative methodologies, such as teaching context, types of educational activities, content courses, pedagogical knowledge, or equivalent measures as more comprehensive indicators.

Collaboration during Mathematical Argumentation

A study by Walter and Barros (2011) explores the emergence of two grounded theories. The first theory, based on mathematics, focuses on the collaborative development of mathematical methods among college-level calculus students. These students collaborated to solve a solid revolution volume problem driven by mathematical problem-solving necessity without prior instruction on solution methods. The second theory arises from a microlinguistic analysis of students' mathematical decisions and their application of warrants in substantive argumentation. This communication aimed to clarify and persuade others of the soundness of their hypotheses and mathematical work.

Kollar et al. (2014) conducted a study to explore the impact of dyadic collaboration on the mathematical reasoning skills of students of mathematics teachers with varying levels of prior achievement. The authors utilized a 2×2 factorial design comprising two different instructional methods: collaboration scripts (with vs. without) and heuristic worked examples versus problem-solving. They found that dyadic collaboration led to significant improvements in mathematical reasoning skills compared to individual problem-solving. The results suggest that collaboration among peers is an effective instructional tool for enhancing mathematical reasoning skills. The study aimed to assess the impact of differentiated scaffolding, synergistic scaffolding, and cognitive overload/over-scripting on math students with different prior achievement levels. The students received a mix of collaboration scripts and heuristic work examples to enhance their mathematical reasoning abilities inside a CSCL environment while tackling math-proof problems. Schwaighofer et al. (2017) conducted four treatment sessions wherein learners collaborated in pairs to work on a single mathematical proof problem per session. Urhan and Zengin (2023) conducted an intervention using the ACODESA method integrated with GeoGebra. The intervention consisted of two 50-minute sessions per week for six weeks, with participants divided into six groups of three students each. Throughout all stages of the ACODESA method, participants worked on computers. The stages included individual work, teamwork, scientific debate, self-reflection, and institutionalisation, all of which were recorded with participants' permission. ACODESA stands for Apprentissage Collaboratif, Débat Scientifique et Autoréflexion in French, which translates to collaborative learning, scientific debate, and self-reflection in English (Hitt et al., 2017).

In all studies within this theme, collaboration was employed to collectively solve mathematical problems and to clarify and convince others of the validity of their conjectures and mathematical work. When students actively engage in producing substantial arguments for presentation and peer rebuttal in classroom contexts, the pursuit of consensus among different solution approaches promotes the development of mathematical depth and reflective refinement of students' mathematical reasoning. All studies suggest that future research should recognise more fully how students employ analogical reasoning in theory construction to effectively convey their understanding of mathematical meanings, concepts, and processes. Assessing the social-discursive component of mathematical argumentation skills should involve a performance-oriented measure in which students actively engage in collaborative argumentation, rather than solely describing how they would participate in such situations. Furthermore, it would be valuable to measure not only content-related learning prerequisites, such as domain-specific prior knowledge, but also more general prerequisites like working memory capacity.

Designing and Teaching for Mathematical Argumentation

The research conducted by Castro et al. (2021) focused on the objective analysis of pre-service teachers' arguments while explaining geometry tasks to both colleagues and students. They design and teach classes for elementary school students, and reflect on their designed activities in discussions. Ledezma et al. (2022) analysed future mathematics teachers' reflections on the process of modelling task design during their studies in school. During the internship period, prospective teachers must design and implement the teaching and learning sequence. The referenced studies extensively examine how the characteristics of didactic-mathematical knowledge (including mathematical, didactic, and meta-didactic mathematical dimensions) are employed in argumentative practice.

Assessment for Mathematical Argumentation

The study conducted by Renninger et al. (2023) introduced the assessment of mathematical comprehension (AMC), an interactive online assessment. In this study, the AMC was used to investigate undergraduate students' comprehension of mathematical argumentation concerning specific textual features and individual learner characteristics. The assessment of mathematical argumentation comprehension included two primary aspects: (1) differentiating metamathematical structures, which measures the ability to identify definitions, theorems, and proofs within the text, and (2) synthesising argument structures, which is defined as the ability to connect various components (such as data, warrant, or claim) within a presented argument. In addition, the study examined the effect of text domain (public or abstract) on the comprehension process and its relationship to individual learner characteristics. These findings suggest that future assessments should consider additional variables, such as representations or imagery, in addition to the text domain to enhance the learning context. Developing specific comprehension skills related to mathematical argumentation, beyond distinguishing metamathematical and summarising argument structures, is also essential.

Conclusion, Limitations, and Future Research

In conclusion, the systematic review of 25 articles examines the definition, task characteristics, and selected themes of studies on mathematical argumentation in higher education. This review provides a clear and objective overview of the current research in the field. Mathematical argumentation is characterised as the process of creating deductive arguments employing valid rules of inference to achieve a solution (Pedemonte, 2007). The competence of mathematical argumentation can be assessed by evaluating the validity of arguments. As an activity, the characteristics of mathematical argumentation in higher education tasks encompass a range of activities, from constructing proofs to collaborating with peers, validating skills in mathematical arguments, designing, explaining, and solving mathematical problems. Some studies directly compared the persuasiveness of different types of mathematical arguments, while others assessed the competence of pre-service teachers in conducting valid proofs.

The selected studies were broadly categorised into four themes: Students' Abilities, Collaboration, Designing and Teaching, and Assessment. These themes shed light on the diverse understandings, collaborative efforts, instructional approaches, and assessment methods prevalent in the study of mathematical argumentation in higher education. These themes significantly contribute to our current understanding of mathematical argumentation in higher education. Each theme offers valuable insights for future research recommendations. To improve students' abilities in this area, it is necessary to further explore the integration of didactic-mathematical knowledge into mathematical argumentation. This will allow student teachers to evaluate the teaching and learning process and make improvements for future implementations (Ledezma et al., 2022). Furthermore, it would be beneficial to examine students' mathematical argumentation skills in connection with their working memory capacity and text comprehension abilities. Text that promotes the triggering and sustaining of interest can encourage learners to continue engaging with mathematics, support the development of more analytical proof schemes, and influence how learners position themselves to read and comprehend mathematical argumentation (Renninger et al., 2023).

This study has limitations because it exclusively examined articles published in journals listed in the Scopus database. Consequently, the findings may not fully represent studies conducted outside the scope of this database. Furthermore, the study's limitation is that it excludes potentially valuable insights from book chapters. This highlights the importance of future research to consider a broader range of literature sources, including books and book chapters, to gain a more comprehensive understanding of the topic. In light of the search keywords used in this study may not have captured all studies related to mathematical argumentation in higher education, as higher education can also be described using alternative terms. Therefore, it is recommended that future research consider the utilisation of other databases and relevant alternative keywords to comprehensively address issues related to mathematical argumentation in higher education.

This study provides valuable insights into the landscape of mathematical argumentation in higher education, offering a comprehensive view of task characteristics, themes, and the attributes of primary articles. The findings highlight the significance of mathematical argumentation as both an activity and a skill in assessing the validity of arguments, proving conjectures, and enhancing problem-solving abilities. In the future, it will become important to recognise the diversity of approaches used in the teaching and assessment of mathematical argumentation through an emphasis on collaborative learning, innovative task design, and comprehensive assessment strategies.

Conflict of Interest

The authors disclose that they have no actual or perceived conflicts of interest. The authors disclose that they have not received any funding for this manuscript beyond resourcing for academic time at their respective universities. The authors have produced this manuscript without artificial intelligence support.

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The primary articles included in the systematic review were highlighted with an asterisk.

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APPENDIX A.

Table A1

Characteristics of Studies in the Primary Articles

No.	Author, Year	Country	Participant	Method	Types of Tasks
1.	(Morris, 2007)	USA	34 pre-service teachers	Qualitative: Individual interviews.	Evaluate students' arguments
2.	(Inglis, Mejia-Ramos., & Simpson, 2007)	United Kingdom	6 postgraduate mathematics students	Qualitative: A series of task-based interviews	Evaluate conditional statements
3.	(Inglis & Mejía-Ramos, 2009a)	United Kingdom	58 undergraduate students from three highly ranked UK universities and 56 research active mathematicians	Quantitative: Experiment	A statement of Young's Inequality
4.	(Inglis & Mejia-Ramos, 2009b)	United Kingdom	194 undergraduate students from three highly ranked British universities and 190 research-active mathematicians	Quantitative: Experiment	Evaluate three different arguments (a heuristic argument, an induction argument, and a visual argument)
5.	(Walter & Barros, 2011)	USA	18 university calculus students'	Qualitative: Eighteen second semester calculus students, comprising four groups, worked collaboratively during 2-h class sessions three times per week on task.	Finding the volume of a solid
6.	(Bleiler et al., 2013)	USA	34 prospective secondary mathematics teachers	Qualitative: designed and implemented a series of five activities.	Validations of students' written proofs
7.	(Kollar et al., 2014)	Germany	101 mathematics teacher students	Quantitative: Experiment	Collaboration scripts and heuristic worked examples
8.	(Vogel et al., 2016)	Germany	101 math teacher students	Quantitative: A 2 x 2 experiment with the factors collaboration script and heuristic worked examples	Proof task and the formal conjectures the learners worked with collaboratively in the three treatment sessions
9.	(Schwaighofer et al., 2017)	Germany	108 university freshmen	Quantitative: 108 university freshmen worked in dyads on mathematical proof tasks in four treatment sessions.	Collaboration scripts and heuristic worked examples

10.	(Demiray & Bostan, 2017)	Turkey	115 pre-service middle school mathematics teachers	Quantitative: A cross sectional survey design	A proof questionnaire containing three proof statements
11.	(Hidayat et al., 2018)	Indonesia	60 pre-service mathematics teachers	Quantitative: Experiment	Determining the derivative of a function's graph
12.	(Nagel et al., 2018)	Germany	86 first-year students in mathematics	Qualitative: Created three open-ended questions which illustrate the thinking process and development of students' argumentations	Proving three theorems in geometry
13.	(Erkek & Bostan, 2019)	Turkey	8 prospective middle school mathematics teachers	Qualitative: Case study	2 geometry tasks on triangles and 2 geometry tasks based on circles
14.	(Zambak & Magiera, 2020)	USA	37 grades 1–8 pre-service teachers'	Quantitative: Experiment	Collective argumentation while solving crypto-arithmetic problems about a multi-digit addition algorithm
15.	(Iwuanyanwu & Ogunniyi, 2020)	South Africa	40 pre-service teachers	Quantitative: Experiment	Conceptual maths-in-physics (MIP) problems in mechanics
16.	(Castro et al., 2021)	Colombia	3 Pre-service teachers'	Qualitative: Pre-service teachers chose to design and to present classes on Euclidian geometry in elementary school.	Explaining geometry tasks
17.	(Simsek, 2021)	Turkey	50 pre-service middle school teachers and Individual interviews with 7 PSMT	Qualitative: Semi-structured individual interviews.	Geometry tasks
18.	(Sommerhoff et al., 2021)	Germany	45 Undergraduate mathematics students	Quantitative: Two approaches were implemented during	Proof construction tasks

20.	(Marchant et al., 2021)	USA	14 prospective secondary mathematics teachers'	Qualitative: semi-structured interviews.	Each participant selected a focus class and a unit of instruction for which they designed and implemented the instruction.
21.	(Vogel et al., 2022)	United Kingdom	167 university students	Quantitative: Experiment	Mathematical conjecturing problem in elementary number theory
22.	(Tristanti & Nusantara, 2022)	Indonesia	70 prospective mathematics teachers	Quantitative: Experiment	Proof problems in pyramid
23.	(Ledezma, et al., 2022)	Spain	3 prospective teachers from the master's program	Qualitative: At the end of the submodule, the prospective teachers must expose a modelling problem (wording of the task, solving process, curricular location of the contents) as a final task.	Designing of a modelling task
24.	(Renninger et al., 2023)	USA	64 undergraduate students	Quantitative: Cluster analysis	The Assessment of Mathematical Comprehension (AMC) from the Math Images wiki page collection
25.	(Urhan & Zengin, 2023)	Turkey	18 university students	Qualitative: An intervention using the ACODESA method integrated with GeoGebra.	A task related to the limit involving sinx/x